CHAPTER THREE

BINARY OPERATION

Operation rules in ordinary Algebra:

These operational rules are

1) Closure:

- A statement is opened when no limitation is placed on it, and closed when a limitation is placed on it.
- For example Kofi is a boy is an opened statement, but Kofi is a boy in the class is a closed statement.
- Also x is a number is an opened statement, but x is a number less than 10 is a closed one.

2) Commutative law:

a) If a + b = b + a, then the given operation which is + or addition is commutative.
b) If a × b = b × a, then the given operation which is × or multiplication is commutative.
c) Lastly if a Δ b = b Δ a, then the given operation which is Δ is commutative.

Q1) Using the numbers 3 and 4, determine whether or not addition is commutative.

N/B L.H.S = Left hand side and R.H.S= Right hand side.
Soln.

For addition (+) to be commutative, then 3+ 4 = 4 + 3.

Consider the L.H.S i.e 3 + 4 =7.

Consider the R.H.S i.e 4+3 =7.

Since L.H.S = R.H.S, then addition is commutative.

Q2) Using the numbers 3 and 4, determine whether or not subtraction is commutative.

Soln.

If subtraction is commutative, then $3 - 4 = 4 - 3$.

Considering the L.H.S, $3 - 4 = -1$.

Considering the R.H.S, $4 - 3 = 1$.

Since the L.H.S $\neq$ R.H.S

i.e L.H.S is not equal to the R.H.S, then subtraction is not commutative.

Q3) Using the numbers 3 and 4, determine whether or not multiplication is commutative.

Soln.

For multiplication to be commutative, then $3\times4 = 4\times3$.

L.H.S = $3\times4 = 12$.

R.H.S = $4\times3 = 12$.

Since L.H.S = R.H.S, then the operation which multiplication is commutative.

3) Associative Law:

a) If $(a + b) + c = a + (b + c)$, then addition is associative.
b) If $(a \times b) \times c = a \times (b \times c)$, then multiplication is associative.
c) If $(a * b) * c = a * (b * c)$, then the operation which is $*$, is associative.

Q1) Using the numbers 2, 3 and 5, determine whether or not addition is associative.

Soln.

If + (addition) is associative, then $(2+3) + 5 = 2 + (3+5)$.

L.H.S = (2+3) +5 = 5+5=10

R.H.S = 2+ (3+5) = 2+8=10

Since the L.H.S = R.H.S, then addition is associative.

Q2) Using 2, 3 and 5, determine whether or not multiplication is associative.

Soln.

If multiplication is associative, then $(2\times3) \times 5 = 2\times (3\times5)$.

L.H.S = $(2\times3) \times5 = 6\times5 = 30$

R.H.S = $2\times (3\times5) = 2\times15 = 30$

Since L.H.S = R.H.S => × (multiplication) is associative.

Q3) Using 2,3 and 5, determine whether or not subtraction is associative.

Soln

If subtraction (-) is associative, then $(5 - 3) – 2 = 5 - (3 - 2)$

L.H.S = $(5-3) –2 = 2 – 2 = 0$

R.H.S = $5 - (3 - 2) = 5 - 1= 4$

Since R.H.S ≠ L.H.S, then (–) or subtraction is not associative.

NB: $a\times (b + c)$ is the same as $a (b + c)$

4)Distributive Law:

- If $a (b + c)$ or $a \times (b + c) = ab + ac$, then multiplication is said to be distributive over addition (+), or multiplication is said to be distributive with respect to addition.

- Also if $a (b \Delta c) = ab \Delta ac$, then multiplication is distributive over Δ, or multiplication is distributive with respect to the operation Δ.

An operation:

* An operation is a symbol, with a given meaning.

* For example if $a*b = 2a+b$, then the symbol * becomes an operation, and $a*b$ means that take twice of a and add it to b.

* Also given that $a \Delta b = a^2 + b^2$, then the symbol Δ becomes an operation, and $a \Delta b$ means add a squared to b squared.

* Other examples of operation which we are familiar with are addition (+), subtraction (-), division (÷) and multiplication (×).

* Lastly any symbol can be used to represent an operation, provided its meaning is given.

The Identity Element:

The identity element of a given operation has no effect on that given operation.

* For example the identity element of addition is 0 (zero), since any number added to zero gives us the same number. (i.e zero has no effect on addition).

* For examples are

$3 + 0 = 3$

$5 + 0 = 5$

$2 + 0 =2$

* The identity element of multiplication is one, since one has no effect on multiplication.

i.e $2×1=2$

$4×1= 4$

$5×1= 5$

* Therefore assume Δ to be a given operation, and if i.e = the identity element of Δ, then

2 Δ i.e = 2

4 Δ i.e = 4

1 Δ i.e = 6

Q1)

Δ	1	2	3	4
1	4	1	7	2
2	6	2	3	3
3	5	3	4	5
4	1	4	1	4

The given table is that for the operation Δ. By making a careful study of it, determine the identity element of the given operation.

Soln.

From the table

1 Δ 2 = 1

2 Δ 2 = 2

3 Δ 2 = 3

4 Δ 2 = 4

=> Any number Δ 2 = that number

=> 2 has no effect on the given operation, and as such it is the identity element.

Q2)

*	2	3	5	7
2	3	1	2	4
3	1	4	3	2
5	7	3	5	6
7	6	2	7	4

The given table is that drawn for a certain operation, which is represented by the symbol *. By careful analysis, determine the identity element for the given operation.

Soln.

A careful study indicates the following:

2 * 5 = 6

3 * 5 = 3

5 * 5 = 5

7 * 5 = 7

- This implies that 5 had no effect on the given operation. Therefore the identity element = 5